Related Topics
Resonance, Q factor, dissipation factor, bandwidth, critical or optimum coupling, characteristic impedance, Pauli method, parallel conductance, band-pass filter, sweep.

Principle
The \( Q \) factor of oscillating circuits is determined from the bandwidth and by the Pauli method. In inductively coupled circuits (band-pass filters) the coupling factor is determined as a function of the coil spacing.

Equipment
1. Digital function generator 13654-99
2. 25 MHz Digital storage oscilloscope 11456-99
3. HF-coil, 35 turns, 75 \( \mu \)H 06915.00
4. HF-coil, 50 turns, 150 \( \mu \)H 06916.00
5. HF-coil, 75 turns, 350 \( \mu \)H 06917.00
6. Coil, 150 turns, short 06520.01
7. Variable capacitor, 500 pF 06049.10
8. Carbon resistor 1 W, 22 kOhm 39104.34
9. Carbon resistor 1 W, 47 kOhm 39104.38
10. Carbon resistor 1 W, 100 kOhm 39104.41
11. Carbon resistor 1 W, 1 MOhm 39104.52
12. Carbon resistor 1 W, 82 kOhm 39104.40
13. Capacitor /case 1/ 470 pF 39105.07
14. Connect. plug white 19 mm pitch 39170.00
15. Connection box 06030.23
16. G-clamp 02014.00
17. Meter scale, demo, \( l = 1000 \) mm 03001.00
18. Adapter, BNC-socket/4 mm plug 07542.27
19. Connecting cord, \( l = 250 \) mm 07360.02
20. Screened cable, BNC, \( l = 750 \) mm 07542.11
21. Screened cable, BNC, \( l = 1500 \) mm 07542.12

Fig. 1: Experimental setup.
**Tasks**

1. Determine the dissipation factor \( \tan \delta \) and the quality factor \( Q \) from the bandwidth of oscillating circuits.

2. Determine the dissipation factor and \( Q \) factor of oscillating circuits from the resonant frequency \( (\omega_0) \), the capacitance \( C_{\text{tot}} \), and the parallel conductance \( G_p \) by the Pauli method.

3. Determine the coupling factor \( k \) and the bandwidth \( \Delta f \) of a band-pass filter as a function of the coil spacing \( s \).

4. Analyze and verify the measurements using the “measure” analysis software.

**Set-up and Procedure**

**Step 1:**
- Connect the U-f-output of the generator to the X-input of the oscilloscope in X-Y operation (channel 1)
- Connect the signal passing through the circuit setup (Fig. 2) to the Y-input of the oscilloscope (channel 2)
- Set the generator function to the \( f_1 \ldots f_2 \) mode and a logarithmic range with adequate sweep factor \( (f_n=f_{n-1}^{\text{sweep factor}}) \)
- Enter the desired values for the frequency range \( (f_1 \text{ and } f_2) \) and choose the \( f_1 \ldots f_2 \) mode from the U-f menu. The menu contains some preset frequency intervals which you can use to quickly change the frequency range you want to use.

A “constant” current is fed to a parallel oscillating circuit consisting of a coil, a capacitor and a high-value resistor \( R_s \) (Fig. 2). First, find the resonance curve with the generator frequency swept and the oscilloscope on X-Y operation. Now vary the frequency manually with the sweep switched off and measure the frequencies \( f_1, f_0 \) and \( f_2 \) (Fig. 3). Figure 4 shows a measured voltage response for the oscillating circuit with the 75 turns coil.

![Fig. 2: Parallel oscillating circuit with series resistor \( R_s \).](image-url)

![Fig. 3: Bandwidth of an oscillating circuit.](image-url)
Step 2:

With sweep on, damp the oscillating circuit with additional parallel resistors $R_z$ (additional conductances $G_z$). Plot the resonance voltage $U_0$ reciprocally against the additional conductance $G_z$.

Step 3:

Construct two identical oscillating circuits as shown in Fig. 5; they should have a fairly large spacing $s$ and be tuned to the same frequency (on sweep) with the variable capacitors. Now bring the two coils closer together (inductive coupling), and measure the voltages of the resonant frequency at the peak point or at the centre and peak points, and the frequencies $f_1$, $f_m$, $f_0$ as a function of the coil spacing $s$ (see Fig. 8).

Fig. 5: Setup of two identical oscillating circuits.
Theory and Evaluation

For the parallel resonance circuit consisting of a dissipative inductance and a capacitance it is possible to postulate an equivalent circuit (Fig. 6) with the parallel conductances $G_{pl}$ and $G_{pc}$. In complex notation, the conductance of the parallel resonance circuit is

$$Y(\omega) = G_{pl} + G_{pc} + i(\omega C - \frac{1}{\omega L})$$  \hspace{1cm} (1)$$

where $i$ is the imaginary unit, $\omega$ is the frequency, $C$ is the capacitance and $L$ is the inductance. At resonance, the reactance components cancel each other out,

$$\omega C_0 = \frac{1}{\omega L_0}$$  \hspace{1cm} (2)$$

and we obtain

$$Y(\omega) = G_{pl} + G_{pc} = G_p$$  \hspace{1cm} (3)$$

with

$$G_p = \frac{1}{Z_k} \tan \delta_k$$

$$Z_k = \frac{1}{\omega_0 C}$$

$$\omega_0 = 2\pi f_0$$  \hspace{1cm} (4)$$

$Z_k$ is the characteristic impedance of the non-dissipative circuit, $\tan \delta_k$ the dissipation factor of the circuit. The quality or $Q$ factor is defined as follows:

$$Q = \tan \delta_k^{-1}$$

**Task 1:** Determine the dissipation factor $\tan \delta_k$ and the quality factor $Q$ from the bandwidth of oscillating circuits

The $Q$ factor is defined in terms of the bandwidth of the oscillating circuit (cf. Fig. 3) as follows:

$$Q = \frac{f_0}{f_2 - f_1}$$  \hspace{1cm} (5)$$
Coupled oscillating circuits

Example:
For the HF coil (75 turns) and the oscillating circuit capacitance \( C_{tot} = C + C_i = 492 \) pF, the exemplary measurements gave:

\[
\begin{align*}
 f_0 &= 380.6 \text{ kHz}, \\
 f_1 &= 378.5 \text{ kHz}, \\
 f_2 &= 382.8 \text{ kHz},
\end{align*}
\]

hence \( Q = 88.5 \) and \( \tan \delta_k = 1/88.5 \)

For the 150 turns coil and \( C_{tot} = 492 \) pF, the measurements gave:

\[
\begin{align*}
 f_0 &= 225.3 \text{ kHz}, \\
 f_1 &= 220.3 \text{ kHz}, \\
 f_2 &= 230.5 \text{ kHz},
\end{align*}
\]

hence \( Q = 22 \) and \( \tan \delta_k = 1/22 \)

It should be noted that the \( Q \) factor measured applies to the whole system, so that the damping by \( R_s \) and \( R_i \) was also included along with the oscillating circuit comprising \( L \) and \( C \).

Task 2: Determine the dissipation factor and \( Q \) factor of oscillating circuits from the resonant frequency \( (\omega_0) \), the capacitance \( C_{tot} \), and the parallel conductance \( G_p \) determined by the Pauli method.

If we plot the resonance voltage \( U_0 \) reciprocally against the additional conductance \( G_z \) we obtain a straight line; the point at which the line on extrapolation intersects the x-axis gives the parallel conductance \( G_p \) (Pauli method, Fig. 7). The characteristic impedance \( Z_k \), dissipation factor \( \tan \delta_k \) and the \( Q \) factor can be calculated from the parallel conductance \( G_p \), the resonant frequency \( \omega_0 = 2 \pi f_0 \) and the oscillating circuit capacitance \( C_{tot} = C + C_i \).

When \( I \) is constant, the equation

\[
\frac{1}{U} = \frac{1}{I} (G_p + G_z)
\]  

(6)

describes a straight line. In the limit case

\[
\frac{1}{U} \to 0
\]  

(7)

we can obtain \( G_p \). We must consider here the conductance values already available, \( \frac{1}{R_s} \) and \( \frac{1}{R_i} \), so that \( \frac{1}{U} \) is plotted against

\[
\frac{1}{G_z} = \frac{1}{R_z} + \frac{1}{R_s} + \frac{1}{R_i}.
\]
**Example**

We determined the resonance voltage $U_0$ and the additional conductance $G_\tau$ for the 75-turn HF coil and the 150-turn coil. From this we obtained two graphs depicted in Fig 7.

From the intersection of the graph with the x-axis we obtained the parallel conductance $G_p = 15.9 \times 10^{-6} \Omega^{-1}$ for the 75-turn coil. Inserting $C_{tot} = 492 \text{ pF}$ and $f_0 = 380.6 \text{ kHz}$ (determined in task 1) in equation 4 gives:

\[
Z_k = 850 \text{ ohms} \\
tan \delta_k = 13.51 \times 10^{-3} \\
Q = 74.
\]

For the 150-turn coil we obtained the parallel conductance $G_p = 37.5 \times 10^{-6} \Omega^{-1}$ and $C_{tot} = 492 \text{ pF}$. With $f_0 = 234.8 \text{ kHz}$ we obtained $Z_k = 1378 \text{ ohms}$, $tan \delta_k = 51.67 \times 10^{-3}$ and $Q = 19.35$.

![Graph](image)

Fig. 7: Reciprocal resonance voltage as a function of the additional conductance, used to determine $G_p$. 1. HF coil, 75 turns; 2. 150-turn coil.

**Task 3: Determine the coupling factor $k$ and the bandwidth $\Delta f$ of a band-pass filter as a function of the coil spacing $s$.**

If two identical circuits (inductive and capacitive) are coupled (Step 3), we obtain a band-pass filter whose resonance behavior greatly depends on the coupling factor $k$. Using the measured values $U_m$ and $U_{max}$ from Fig. 8 we can calculate the coupling factor $k$ in accordance with

\[
k = \tan \delta_k \left[ \frac{U_{max}}{U_m} \pm \sqrt{\left( \frac{U_{max}}{U_m} \right)^2 - 1} \right] \quad (8)
\]

where $\tan \delta_k$ is the dissipation factor of the single circuit.

![Diagram](image)

Fig. 8: Resonance curves for a band-pass filter at different degrees of coupling.
The stronger the coupling, the further apart the peak point frequencies are. The bandwidth $\Delta f$ of the supercritical filter is determined by the frequencies at which the voltage has fallen to the voltage $U_m$ of the centre frequency $f_m$ and can be calculated from the peak point frequencies $f_1$ and $f_2$ in accordance with

$$\Delta f = \sqrt{2}(f_2 - f_1)$$

(9)

**Example**

Figures 9 and 10 depict the results of our exemplarily measurement.

**Note**

Coupled mathematical pendulums provide a mechanical analogy to coupled oscillating circuits (the two natural frequencies of the coupled pendulums, which produce beats, correspond to the two peak point frequencies).
Data analysis

The measurement results can be plotted and analyzed with the “measure” software. To enter the data manually, choose Enter data manually from the Measurement menu (Fig. 11).

Choose the adequate number of channels (for example, if you measured voltage vs. frequency, choose 2 channels, if you want to plot two curves on the same plot, choose 3 channels, etc.). The x-data field creates integer numbers of the specified length (number of values) starting with 1. Enter the number of values that corresponds to the number of measured data points, and enter the Title, Symbol, Unit and Digits for each channel. Note: you can adjust these values later, as well as specify which channel is plotted on the x and y axes. When done, click Continue.

Fig. 11: Enter data manually option in the “Measure” software.

Fig. 12: Entering the data in the “Measure” software.
A table opens, where you can input the measurement values for each channel (Fig. 12). You can add and remove measurement points using the symbols at the bottom of the table. Click OK when done. A plot of the data appears.

Go to Channel manager in Measurement menu to specify the data channels to be plotted as the x and y axes. In the left column you can see the data channels you entered. The right column will be the new arrangement of the x and y axes. Notice the field highlighted in blue in the right column (default is the Destination x-axis field, but you can change that by double-clicking on a required field). Mark a data channel in the left column, and then click on the transfer arrow between columns. The channel now appears in the highlighted field in the right column. Hence, to create plots of frequency vs. amplitude ratio, choose frequency as the new x-axis, and amplitude as the y-axis. Click OK when done.

You can easily access the data table and display options by clicking on the plot with the right mouse button. For example, to obtain a plot with logarithmic scale on the x-axis, go to display options, choose the X-data tab, and change scaling to logarithmic. You can adjust other display options, like the x and y ranges and labels etc.

It is recommended you save the measurement after you enter the data, you can do that at any time from the File menu. You can load saved data with the Open experiment command in the File menu.

Note: many commands are also accessible directly through corresponding icons located in the taskbar.

To analyze the plotted data, scroll down the Analysis menu and choose the required option, for example Show extrema. A window appears, showing the extrema of the plot (Fig. 13). Mark the Visualize results option to display the values on the plot.

You can export a plot to a file using the Export data option from the Measurement menu. Mark Save to file and Export as bitmap to save the plot as a bitmap image.